Autoresonant propagation of incoherent light-waves

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Abstract: We study, theoretically and experimentally, the evolution of optical waves with randomly-fluctuating phases in a spatially chirped nonlinear directional coupler. As the system crosses its linear resonance, we observe collective self-phase-locking (autoresonance) of all mutually-incoherent waves, each with its own pump, and simultaneous amplification until the pumps are exhausted. We show that the autoresonant transition in this system exhibits a sharp threshold, common to all mutually-incoherent waves comprising the light beam.

OCIS codes: (190.5940) Self-action effects; (190.4420) Nonlinear optics, transverse effects in.

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1. Introduction

Amplification of optical waves in weakly-coupled nonlinear systems requires phase matching. Examples range from coupled waves in a nonlinear directional coupler [1], and high harmonics generation [2], to wave mixing in nonlinear photonic lattices [3], and more. Without phase matching, as the waves propagate, power flows back and forth between the driving and driven waves. In many cases, phase matching can be achieved via anisotropy in the nonlinear medium (birefringence), or by modulating the medium (periodic poling [4] or grating-assisted phase matching [2,5]). Recently, several papers suggested adiabatic processes in nonlinear optical systems for efficient unidirectional power transfer, e.g., for efficient sum frequency generation [6], stimulated Raman adiabatic passage between three waveguides [7,8] and adiabatic passage of light via continuum [9]. In all these processes, the evolution of the system is highly sensitive to the intensity, and as the involved intensities increase - the efficiency decreases. This raises an immediate question: how can one amplify the driven wave deep into the nonlinear regime, reaching very high intensities? Would it be possible to enable efficient unidirectional power transfer in spite the fact that, in the presence of nonlinearity, the system parameters (e.g., coupling strength, chirp rates) vary during propagation? Is there a way to assure efficient amplification and unidirectional power flow for a wide range of physical parameters (optical wavelength, chirp rate, light intensity)? The answer is yes, this is indeed possible, through a phenomenon called autoresonance. Recently, we have shown that autoresonance, a nonlinear phenomenon in which phase-locking and amplification are automatically maintained, can yield efficient amplification of optical waves [10]. Autoresonant amplification arises from the tendency of a nonlinear system to remain in resonance with an external modulation, despite variations in the system parameters. Autoresonant evolution of waves involves adiabatic passage through a linear resonance of the system, automatic phase-locking above a sharp threshold, and unidirectional power flow from one wave to the other - resulting in amplification to predetermined values [11,12]. Autoresonance is a fundamental nonlinear process taking place in many nonlinear systems, ranging from plasmas [13], particle accelerators [14] and fluidic systems [15] to solitons [16], planetary dynamics [17], BEC condensates [18], superconducting Josephson junctions [19] and more.

Autoresonance has always been traditionally studied in highly coherent systems, because the phase-mismatch plays such a crucial role in the process. However, optics presents an opportunity to study wave dynamics for any degree of coherence – from fully coherent to

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completely uncorrelated waves. In terms of the physics involved, an incoherent wave system is generically a multi-wave system. As such, studying autoresonance with incoherent (or partially coherent) waves raises a series of fundamental questions: would this multi-wave system exhibit a collective behavior, with a single, well defined, autoresonance threshold, or would each wave (which is uncorrelated or partially-correlated with the other waves) pass its own threshold individually? Would the coherence of the waves be affected by the highly nonlinear autoresonance phenomenon? These are just few of the many questions that come up when autoresonance is studied in conjunction with incoherent waves.

Here, we analyze the autoresonant evolution of mutually-incoherent (random-phase) optical waves. We investigate the nonlinear dynamics of a incoherent optical beam in a directional coupler: a beam comprising of multiple waves, each having its own randomly-fluctuating phase. We show that, as the system crosses a linear resonance, the phases of all waves lock together simultaneously, yielding a continuous power flow from the incoherent beam in one waveguide (the driving beam) to the incoherent beam in the other (the driven beam), in a unidirectional fashion, until all the power in the driving wave is exhausted [We emphasize that this process is reversible, when the chirp direction is reversed, since the refractive index is purely real; likewise, when one places a mirror at the output plane the unidirectional power transfer is reversed]. We find that this process is a collective phenomenon, having a common threshold, controlled by the total intensity of the beam, where all stochastically-fluctuating waves phase-lock to their respective pumps together. We show that, being a collective phenomenon, the intensity of just one of the waves controls the dynamics of all the mutually-incoherent waves simultaneously. That is, varying the intensity of a single wave – affects the autoresonant dynamics of all waves in the system, even though they are mutually uncorrelated.

2. Theory

We begin with the (1+1)D dimensionless nonlinear paraxial nonlinear Schrodinger-type wave equation (NLSE) describing the propagation of a monochromatic optical wave of a slowly-varying envelope ψ in a fixed refractive index structure $\Delta n_L(x,z)$ and in the presence of nonlinearity:

$$i\,\partial\psi/\partial z = \partial^2\psi/\partial x^2 + \left[\Delta n_L(x,z) + \Delta n_{NL}(x,z)\right]\psi \ . \tag{1}$$

Here $\Delta n_L(x,z)$ varies in both the longitudinal (z) and the transverse (x) directions, and $\Delta n_{NL}(x,z)$ is the refractive index change induced by ψ through the nonlinearity. We are interested in temporally-incoherent light, hence we assume that the wavepacket is composed of many waves which are mutually incoherent, that is, their modal amplitudes and phases vary randomly in time [20,21]. The incoherent wavepacket can be expressed as $\psi(x,z,t) = \sum_n \exp\left(i\,\varphi_n(t)\right)\phi_n(x,z)$, where $\varphi_n(t)$ is a randomly fluctuating phase, and $\phi_n(x,z)$ is the normalized wavefunction of the *n*th wave. Substituting $\psi(x,z,t)$ into Eq. (1), and separating into (mutually-uncorrelated) fields, we find that each (time-independent) wave amplitude obeys a separate NLSE:

$$i\,\partial\phi_n/\partial z = \partial^2\phi_n/\partial x^2 + \left[\Delta n_L(x,z) + \Delta n_{NL}(x,z)\right]\phi_n \ . \tag{2}$$

We concentrate on systems where the response time of the nonlinearity of the medium is much slower than the typical fluctuations time [20,21]. In such systems, the nonlinear refractive-index changes are proportional to time-averaged intensity structure of the wavepacket: $\left\langle |\psi|^2 \right\rangle = \sum_n |\phi_n(x,z)|^2$. That is, the nonlinear changes are proportional to the sum of the intensities of the mutually-uncorrelated waves, because the contributions of the interference terms vanish due to the averaging over the random phases [20,21]. For concreteness, we study nonlinearity of the Kerr type, which describes, for example, the

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photorefractive nonlinearity (saturable nonlinearity [22,23]) at low intensity ratios. In this case the nonlinear refractive index change follows $\Delta n_{NL}(x,z)=\sum_n |\phi_n(x,z)|^2$ which yields n equations, coupled through the intensities, describing the evolution of the mutually-incoherent waves:

$$i\,\partial\phi_n/\partial z=\partial^2\phi_n/\partial x^2+\left[\Delta n_L(x,z)+\sum_n |\phi_n(x,z)|^2\right]\phi_n\ . \quad (3)$$

This system of equations is in fact a Manakov-type system with an additional external potential, $\Delta n_L(x,z)$. This potential (fixed refractive index structure) $\Delta n_L(x,z)$ is made up of two coupled waveguides: the left waveguide does not change during propagation, whereas the depth of the right waveguide varies linearly with z (i.e., it is spatially-chirped). The linear refractive index profile [Fig. 1a], $\Delta n_L(x,z)=\Delta n_1(x)+\Delta n_2(x)g(z)$, is the sum of a propagation-invariant function [$\Delta n_1(x)$], which has the profile of a symmetric directional coupler, and an evolving part [$\Delta n_2(x)g(z)$], which is the product of a function that has the profile of the right waveguide only [$\Delta n_2(x)$] and a linear function of z [$g(z)=\alpha z$; α being the spatial chirp rate]. In this setup the waves experience linear resonance when the index profiles (linear + nonlinear) of the two waveguides are identical. The coupling between the two waveguides is most efficient at resonance, and decreases dramatically when they are detuned from one another [4]. The two waveguides are weakly coupled, thus allowing us to use the coupled mode (perturbation) theory. Assuming a solution of the form: $\phi_n(x,z)=\exp(i\beta_n z)\left[c_{R,n}(z)u_{R,n}(x)+c_{L,n}(z)u_{L,n}(x)\right]$, where $u_{R,n}$ ($u_{L,n}$) is the normalized eigenmode of the right (left) waveguide of the nth wave without nonlinearity and chirp. $c_{R,n}$ and $c_{L,n}$ are the complex amplitudes of $u_{R,n}$ and $u_{L,n}$ respectively, and $\beta_n=\int_{-\infty}^{\infty}u_{R,n}^{*}H_0\,u_{R,n}\,dx$ is the propagation constant of the nth wave in a single unperturbed linear waveguide, where $H_0=\partial^2/\partial x^2+\Delta n_1(x)$ is the unperturbed Hamiltonian. Here we restrict ourselves to mutually-uncorrelated waves with an identical profile, all having the shape of the highest eigenmode of the unperturbed single waveguide. Substituting ϕ_n into Eq. (3), results in two coupled equations that describe the dynamics of $c_{R,n}$ and $c_{L,n}$, which, in turn, are coupled dynamically to the other waves through the nonlinearity:

$$i\frac{dc_{R,n}}{dz}=\kappa\, c_{L,n}+\chi\left(\sum_m\left|c_{R,m}\right|^2\right)c_{R,n}+\Lambda_0 c_{R,n}z, \quad (4a)$$

$$i\frac{dc_{L,n}}{dz}=\kappa\, c_{R,n}+\chi\left(\sum_m\left|c_{L,m}\right|^2\right)c_{L,n}, \quad (4b)$$

where $\chi=\int_{-\infty}^{\infty}\left|u_L\right|^4 dx$ is the nonlinearity strength, $\kappa=\int_{-\infty}^{\infty}u_R^{*}H_0u_L dx$ is the linear coupling parameter between the right and left waveguides, and $\Lambda_0=\alpha\int_{-\infty}^{\infty}\left|u_R\right|^2\Delta n_2(x)dx$ is the effective chirp rate in the right waveguide. We launch the incoherent beam into the right waveguide (only), and thus set our initial conditions to $\left|c_{R,n}(0)\right|>0$ and $c_{L,n}(0)=0$. Defining a fractional population difference $R_n=\dfrac{\left|c_{L,n}\right|^2-\left|c_{R,n}\right|^2}{\left|c_{L,n}\right|^2+\left|c_{R,n}\right|^2}$ and phase-mismatch, $\Phi_n=\theta_{R,n}-\theta_{L,n}$, where $\theta_{R,n}$ and $\theta_{L,n}$ are the phases of $c_{R,n}$ and $c_{L,n}$, yields two coupled equations:

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$$\frac{dR_n}{dz} = -2\kappa \sqrt{1-R_n^{\ 2}}\,\sin(\Phi_n), \tag{5a}$$

$$\frac{d\Phi_n}{dz} = \Lambda_0 z - \chi\left(\sum\nolimits_m \left| c_{R,m}\left(0\right)\right|^2 R_m\right) + 2\kappa \frac{R_n}{\sqrt{1-R_n^{\ 2}}}\cos(\Phi_n). \tag{5b}$$

The most important feature of these equations is their coupling to the other sets (of $m \neq n$), through the population difference. If one wave transfers power from one waveguide to the other, its R_n changes, and affects the evolution of the other waves. But, since in this system there is no power transfer between uncorrelated waves [20,21], the initial amplitude of each wave is a constant of motion: $\left| c_{R,n}(0)\right| = \sqrt{\left| c_{R,n}(z)\right|^2 + \left| c_{L,n}(z)\right|^2}$.

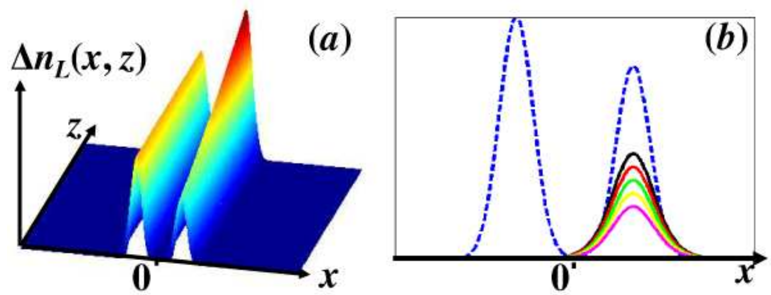

Fig. 1. (a) Linear refractive index profile. (b) Transverse profile of the refractive index at the input face [blue dashed line], along with the intensity profiles of five mutually-incoherent waves (solid lines), at the input face of the directional coupler. Each wave has different initial intensity. All the waves are launched into the right waveguide, where they form the incoherent beam.

We first study the evolution of the waves in the chirped nonlinear coupler through numerical simulations of Eqs. (4a) and (4b). As an example, we study the dynamics of five waves [solid lines in Fig. 1b], each with different initial amplitude $c_{R,n}(0)$, launched into the right waveguide [the cross-section of the waveguide structure at the input plane is marked by blue dashed line in Fig. 1b]. As the waves cross the linear resonance [vertical black dashed line in Fig. 2a], the total intensity in the left waveguide (sum of the amplitudes squared; $\sum\nolimits_m \left| c_{L,m}\right|^2$) suddenly increases [blue solid line in Fig. 2a], at the expense of the amplitudes in the right waveguide [red dashed line in Fig. 2a], until all the power is transferred from the right waveguide to the left one. During this process, each of the uncorrelated waves is amplified to different final amplitude, at the expense of its parent wave, as shown in the inset of Fig. 2a. This suggests that individual phase-locking occurred between each pair of driving and driven waves. However, the whole system remains incoherent, because the mutually-uncorrelated waves are still uncorrelated, in spite of the individual phase-locking within each of its constituents. The fact that phase-locking occurs for all uncorrelated waves at the same position (the location of the linear resonance) is a first indication that autoresonance with incoherent waves is a collective phenomenon, as we prove below.

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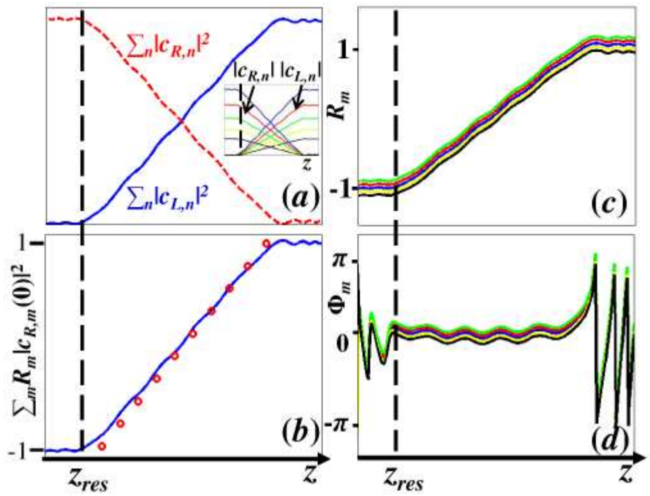

Fig. 2. (a) Evolution of the sum of the squares of the absolute amplitudes in the left [blue solid line] and in the right [red dashed line] waveguides. The inset shows the absolute value of the amplitudes in the right and left waveguides during propagation. As the system crosses the linear resonance [marked in vertical black dashed line], the wave amplitudes in the left waveguide increase, at the expense of the amplitudes in the right waveguide. (b) Propagation of the sum of the population differences [blue solid line], and the theoretical curve, $\sum_m \left| c_{R,m}(0) \right|^2 R_m \approx \Lambda_0 z/\chi$, [red circles]. The efficient amplification results in the flip of the population difference from −1 to 1. (c) Evolution of the population difference for each wave. (d) Evolution of the phase mismatch along *z*, for each wave. In (c) and (d) the plots are slightly diverted, to demonstrate that the dynamics of all waves is identical.

Next, we simulate the evolution of R_n and Φ_n by propagating Eqs. (5a) and (5b). As shown in Fig. 2b by the blue line, as the system crosses the resonance point, the sum of the population difference of all the waves, each multiplied by its corresponding initial intensity [$\sum_m \left| c_{R,m}(0) \right|^2 R_m$], is amplified. Figures 2c and 2d show an even more interesting picture. For all the waves, the population difference, R_n, and the phase mismatch, Φ_n, evolve in exactly the same fashion (the plots where slightly diverted to show that the evolution is indeed identical). For each wave, R_n increases until it flips from −1 to 1. The phase-mismatch for each wave oscillates in exactly the same fashion, around zero, and when the power is completely transferred from the right waveguide to the left - the phases get out of locking. The explanation for this relies on the initial conditions of the waves. When the initial conditions for all the uncorrelated waves are the same, that is $R_n = R_m$ and $\Phi_n = \Phi_m$ for any $n \neq m$, then all waves evolve in identical fashion. The reason for the identical evolution is that the dynamics of each wave is dictated by the same set of two coupled equations [Eqs. (5a) and (5b)], with the same initial conditions. Therefore, the population difference and the phase mismatch for all the waves are equal throughout propagation, and we can write $R_n = R$ and $\Phi_n = \Phi$ for all *n*. We now define the total intensity $I_0 = \sum_m \left| c_{R,m}(0) \right|^2$ which yields two coupled equations that govern the evolution of all the waves:

$$\frac{dR}{dz} = -2\kappa\sqrt{1-R^2}\sin(\Phi), \tag{6a}$$

$$\frac{d\Phi}{dz} = \Lambda_0 z - \chi I_0 R + 2\kappa\frac{R}{\sqrt{1-R^2}}\cos(\Phi). \tag{6b}$$

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Using this form of equations, one can use the autoresonance theory to predict the evolution of the system. The resonance crossing point, shown in Fig. 2, will be at $z_{res} = -\frac{\chi I_0}{\Lambda_0}$ [10]. As the system crosses the resonance - the phase mismatch locks around zero, and power flows from the right waveguide to the left in a unidirectional fashion, to maintain the resonance. This allows us to predetermine the evolution. As shown in [10], the autoresonant evolution of *R* is $R \approx \Lambda_0 z / (\chi I_0)$. We plot this evolution on Fig. 2b in red circles, showing that the population difference indeed evolves according to the autoresonance theory. The process ends only when all the power has transferred from the right to the left waveguide. This evolution describes an ensemble of uncorrelated waves, which are all propagating together. These waves are amplified simultaneously, until all the power has been transferred from one waveguide to the other, and the phases of all these waves exhibit identical evolution.

One of the characteristics of autoresonant dynamics is the sharp threshold on the intensity of the pump for the autoresonant transition. Following [10], we define a dimensionless threshold parameter $T = 2.45\kappa(\chi I_0)^{1/2}\Lambda_0^{-3/4}$. For $T > 1$ we obtain efficient autoresonant amplification, whereas for $T < 1$ the phases do not lock and the amplification is inefficient. In our previous work [10], the threshold for the autoresonant evolution was controlled by the intensity of the wave. In the present experiments, we can control the intensity of each uncorrelated beam independently. This manifests control over the threshold parameter of all the waves by changing the intensity of just one of the uncorrelated waves, say, the *m*-th wave [$|c_{R,m}(0)|^2$]. This collective phenomenon arises from the nonlinear coupling between the waves, which makes the threshold controlled by the total intensity, I_0, which in turn can be controlled by the intensity of each individual wave separately. For example, we can have many uncorrelated waves - each with its own intensity much lower than required for passing the threshold, but the total intensity is above the threshold, hence all the waves are simultaneously amplified. To demonstrate this, we simulate Eq. (1), using a standard beam propagation code. We launch an incoherent beam, composed of five waves [as shown in Fig. 1b] with randomly fluctuating phases into the right waveguide, and we plot their dynamics in Fig. 3. Each wave has amplitude much lower than that needed to cross the threshold for autoresonant amplification. When the total intensity, I_0, is such that $T < 1$, as the system passes the linear resonance some power flows from the right to the left but most of the power still remains in the right waveguide, as shown in Fig. 3a [plotted for $T = 0.95$]. This is basically because the rate of nonlinear variations in the left waveguide is not fast enough to follow the changes in the right waveguide due to the chirp, so the system cannot maintain the resonance. Figure 3b shows the evolution of the amplitudes of the five waves in the right and left waveguides. Clearly, as the system crosses the resonance, some power tunnels, but most of the power remains in the right waveguide. This result is due to inability to lock the phases throughout the propagation as shown in Fig. 3c [the phase-mismatch of each wave is plotted, with some offset between them]. As a result, the population difference for each wave does not flip from -1 to 1 in this case [Fig. 3d]. Altogether, below the threshold the amplification is inefficient for each of the waves.

Next, we slightly increase the amplitude of just one of the waves, so that the threshold parameter increases to slightly above 1 [$T = 1.05$]. Now, as the system crosses the linear resonance, all phases lock simultaneously, all the waves are amplified to their predetermined values, and all the optical power exits the system from the left waveguide - as shown in Fig. 3e.

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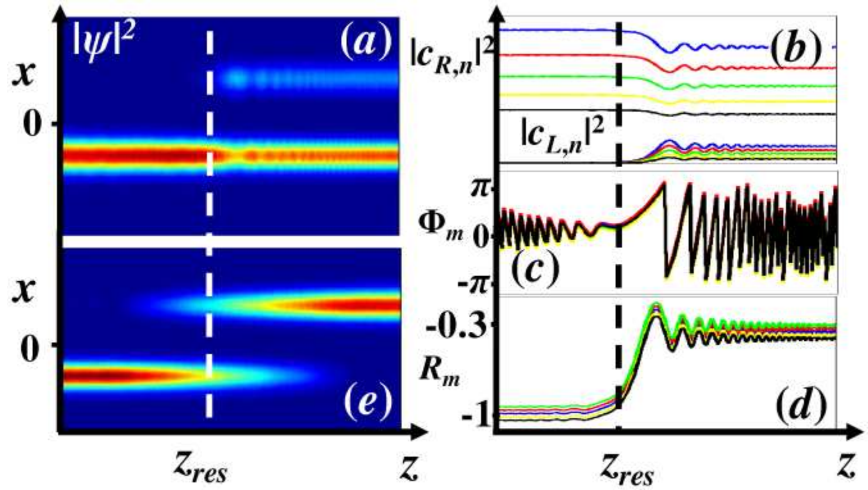

Fig. 3. (a) Propagation of the total intensity of the incoherent beam below the threshold for autoresonant phase locking. (b) Evolution of the intensity and (c) the phase mismatch of each wave below the threshold. As the system crosses the linear resonance [marked in vertical black dashed line], the phases do not lock and the power transfer is inefficient. (c) Evolution of the population difference below threshold. Since the coupling is inefficient, the population differences do not flip from −1 to 1. (e) Propagation dynamics of the total intensity of the incoherent beam, above the threshold for autoresonant phase locking. Now, as the system crosses the linear resonance [vertical white dashed line], the phases lock and the waveguides efficiently exchange power.

3. Experiments

Finally, we study experimentally the autoresonant evolution of incoherent waves. We use the photorefractive screening nonlinearity [22,23], and the induction technique to induce the chirped coupler in a 1.2 cm long SBN:75 crystal [24,25]. The nonlinear index change, $\Delta n_0=1/2n_0^3r_{33}E=0.0008$, arises from the electro-optics effect, E=1000Vcm^{-1} is an external applied field, n_0=2.35 is the linear refractive index of the medium, and $r_{33}\approx$1200pmV^{-1} is the relevant electro-optic coefficient. The chirped directional coupler is induced by the superposition of two mutually-uncorrelated ordinarily-polarized Gaussian beams. Each beam has a 10μm FWHM and they are separated by ~16μm. The chirped waveguide is created by passing one of the beams through a gradient-intensity mask (see details in [10]). Next, we prepare the incoherent beam. We split a 1D 10μm FWHM extraordinarily-polarized beam Gaussian into three beams. We reflect each beam from a piezoelectric mirror driven by a fast oscillating voltage source. By doing so, the phases of the reflected beams oscillate (much faster than the response time of the nonlinearity, ~0.1sec [20]) in an uncorrelated manner, rendering the beams mutually incoherent. We make sure that these beams are indeed mutually incoherent by examining their interference on a camera with a response time somewhat faster than the nonlinearity, 0.01 sec. As the voltage sources driving the piezoelectric mirrors are turned on, the visibility of the interference fringes drops to zero, indicating complete loss of mutual phase correlation. We then pass each beam through a variable attenuator, facilitating control over the intensity of each beam. Finally, we combine the three beams, creating a single temporally-incoherent beam.

We launch the incoherent beam into the right (chirped) waveguide and study the dynamics. Figure 4 shows intensity profiles at the exit plane from the directional coupler. In this setup, the threshold normalized intensity for autoresonant evolution is $I_0 \approx 0.7$. For such intensities, the nonlinearity is not saturated yet, and yields results that are very similar to the Kerr nonlinearity case. For higher intensities, when the saturable nonlinearity becomes saturated, autoresonant evolution still occurs, in the same vein as for the Kerr nonlinearity, but the threshold value and the propagation dynamics of the waves are different than in Kerr media. As expected from theory, for $T<1$, as the system crosses the linear resonance - power tunnels to the left waveguide but most of the power exits the system from the right waveguide, as shown by the blue solid line in Fig. 4a. When $T>1$, when crossing the (linear) resonance - the phases lock, and the power tunnels to the left waveguide efficiently, as shown by red dashed line in Fig. 4a. The results shown in Fig. 4a are obtained by controlling the intensity of just one of the three mutually-incoherent waves. Figures 4b and 4c display the intensity of each wave, below and above the threshold, respectively. For concreteness, in this particular experiment we vary the intensity of the wave with the medium intensity (marked in green dashed line). However, these results are fully reproduced by varying the intensity of each of the other waves. That is, controlling the intensity of just one of the mutually-uncorrelated waves controls the power-transfer process. In all such experiments, when the total intensity is below threshold, each wave passes some power to the left waveguide, but also maintains considerable amount of power within the right waveguide [Fig. 4b]. However, as the total intensity crosses the autoresonant threshold (whether this is done by increasing the intensity of just one – or more - of the mutually-incoherent waves), most of the power launched into the right waveguide exits from the left waveguide [Fig. 4c]. The dynamics is exactly the same for each wave, thereby proving that autoresonance with mutually-incoherent waves is indeed a collective phenomenon.

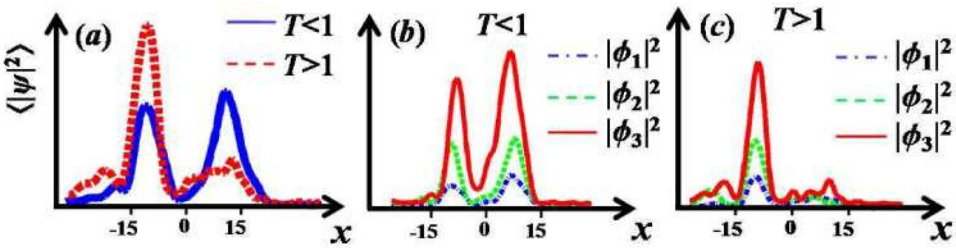

Fig. 4. Experimental results, displaying beam profiles taken at the exit face of the directional coupler. (a) Total intensity below [blue solid line] and above [red dashed line] the threshold. The threshold parameter is controlled by only one of the (mutually-uncorrelated) waves. Above the threshold, sharply, all the power transfers to left waveguide. (b) Each wave below the threshold. (c) Each wave above the threshold. (b) and (c) were obtained by decreasing the intensity of the medium intensity wave [green dashed line] only. The same result is obtained by varying the intensity of the other waves. This shows that the dynamics is indeed collective.

4. Summary

In conclusion, we have studied the impact of autoresonance on the evolution of mutually-incoherent waves. We showed that the evolution of many such waves in a chirped directional coupler results in a collective autoresonance phenomenon, mutual phase locking and efficient simultaneous amplification. Introducing the phenomenon of autoresonance to the area of nonlinear incoherent waves (stochastic nonlinear waves) suggest many new directions and brings up many intriguing questions. For example, it has been shown that autoresonance can be used to excite coherent soliton structures by weak forces [16]. It might be possible to use the same autoresonance techniques to excite incoherent solitons [20,21] by using several external weak forces which are mutually incoherent. By tailoring the parameters of the forces, one might be able to simultaneously excite several localized structures that together comprise an incoherent soliton. Also, several years ago, our group has demonstrated that modulation instability (spontaneous pattern formation) with white incoherent light is a collective phenomenon, exhibiting a single threshold for the entire spectrum [26,27]. Is autoresonance of incoherent wavepackets comprising of different modes (different spatial profiles) also possible? If yes, will it be a collective phenomenon? This brings up an even more intriguing question: is it possible to utilize autoresonance to increase the spatial coherence of waves propagating in a waveguide (external potential) via unidirectional mode conversion? A decade-old study [28] has proposed incoherent solitons as a vehicle to control the spatial coherence, but experiments did not follow – simply because the predicted effects were considered weak. In principle autoresonace could give rise to large coherence increase effects, because it optimizes power transfer processes. Other opportunities lie in the temporal domain. It was already shown [29–31] that nonlinear three wave interactions can yield coherent temporal structures from incoherent ones. One can now envision other exciting possibilities, where the temporal coherence of mutually-uncorrelated waves is increased by autoresonance techniques in an instantaneous medium by the phase-locking mechanism which might create temporal synchronization between uncorrelated waves. We emphasize that the results and ideas presented here are general, relevant to any system supporting stochastic nonlinear waves, in optics and beyond. Examples range from matter waves (BEC) in the presence of thermal cloud [32], incoherent spin waves in magnetic films [33], and more.